Form 5: Chapter 15 (Vectors)
SPM Practice
Fully-Worked Solutions

Paper 1

1 (a) Since $P = (4, 2)$, $\overrightarrow{OP} = \begin{pmatrix} \mathbf{4} \\ \mathbf{2} \end{pmatrix}$.

(b) Since $Q = (-6, 3)$, $\overrightarrow{OQ} = -6\underline{i} + 3\underline{j}$.

Hence, $\overrightarrow{QO} = -\overrightarrow{OQ} = \mathbf{6\underline{i} - 3\underline{j}}$.

2 $\underline{r} = 2\underline{p} - 3\underline{q}$

$\underline{r} = 2(3\underline{a} + 2\underline{b}) - 3(3\underline{a} - \underline{b})$

$\underline{r} = 6\underline{a} + 4\underline{b} - 9\underline{a} + 3\underline{b}$

$\underline{r} = -3\underline{a} + 7\underline{b}$

But it is given that $\underline{r} = h\underline{a} + (h + k)\underline{b}$.

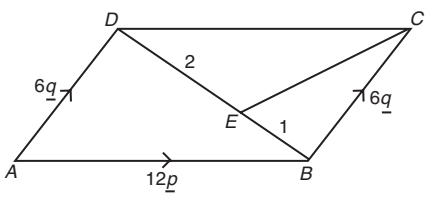
By comparison,

$h = \mathbf{-3}$ and $h + k = 7$

$-3 + k = 7$

$k = \mathbf{10}$

3



(a) $\overrightarrow{BD} = \overrightarrow{BA} + \overrightarrow{AD}$

$= \mathbf{-12\underline{p} + 6\underline{q}}$

(b) $\overrightarrow{EC} = \overrightarrow{EB} + \overrightarrow{BC}$

$= \frac{1}{3}\overrightarrow{DB} + \overrightarrow{BC}$

$= \frac{1}{3}(-\overrightarrow{BD}) + \overrightarrow{BC}$

$= \frac{1}{3}(12\underline{p} - 6\underline{q}) + 6\underline{q}$

$= 4\underline{p} - 2\underline{q} + 6\underline{q}$

$= \mathbf{4\underline{p} + 4\underline{q}}$

4 (a) $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

$= 3\underline{i} + 17\underline{j} - (-2\underline{i} + 5\underline{j})$

$= \mathbf{5\underline{i} + 12\underline{j}}$

(b) $\left|\overrightarrow{AB}\right| = \sqrt{5^2 + 12^2} = \sqrt{169} = 13$

Unit vector in the direction of $\overrightarrow{AB}$

$= \frac{1}{\left|\overrightarrow{AB}\right|}(\overrightarrow{AB})$

$= \frac{1}{13}(5\underline{i} + 12\underline{j})$

$= \mathbf{\frac{5}{13}\underline{i} + \frac{12}{13}\underline{j}}$

5 $\overrightarrow{AB} + 2\overrightarrow{BC} = 11\underline{i} - 13\underline{j}$

$(\overrightarrow{OB} - \overrightarrow{OA}) + 2(\overrightarrow{OC} - \overrightarrow{OB}) = 11\underline{i} - 13\underline{j}$

$$\begin{pmatrix} 5 \\ 3 \end{pmatrix} - \begin{pmatrix} -1 \\ 7 \end{pmatrix} + 2\left[\begin{pmatrix} m \\ p \end{pmatrix} - \begin{pmatrix} 5 \\ 3 \end{pmatrix}\right] = \begin{pmatrix} 11 \\ -13 \end{pmatrix}$$

$$\begin{pmatrix} 6 \\ -4 \end{pmatrix} + 2\begin{pmatrix} m-5 \\ p-3 \end{pmatrix} = \begin{pmatrix} 11 \\ -13 \end{pmatrix}$$

$$\begin{pmatrix} 6 + 2m - 10 \\ -4 + 2p - 6 \end{pmatrix} = \begin{pmatrix} 11 \\ -13 \end{pmatrix}$$

$$\begin{pmatrix} 2m - 4 \\ 2p - 10 \end{pmatrix} = \begin{pmatrix} 11 \\ -13 \end{pmatrix}$$

Equating the x-components:

$2m - 4 = 11$

$m = \mathbf{7\frac{1}{2}}$

Equating the y-components:

$2p - 10 = -13$

$p = \mathbf{-1\frac{1}{2}}$

6 (a) $\overrightarrow{OQ} = \begin{pmatrix} \mathbf{9} \\ \mathbf{-12} \end{pmatrix}$

(b) $\left|\overrightarrow{OQ}\right| = \sqrt{9^2 + (-12)^2}$
$= 15$

Hence, the unit vector in the direction of
$\overrightarrow{OQ} = \frac{1}{15}\begin{pmatrix} 9 \\ -12 \end{pmatrix}$
$= \begin{pmatrix} \mathbf{\frac{3}{5}} \\ \mathbf{-\frac{4}{5}} \end{pmatrix}$.

7 $\overrightarrow{BM} = \overrightarrow{BA} + \overrightarrow{AM}$
$= \overrightarrow{CO} + \frac{1}{2}\overrightarrow{AO}$
$= (4\underline{i} - 2\underline{j}) + \frac{1}{2}(-2\underline{i} - 6\underline{j})$
$= 4\underline{i} - 2\underline{j} - \underline{i} - 3\underline{j}$
$= \mathbf{3\underline{i} - 5\underline{j}}$

8 (a) $A(6, 3) \Rightarrow \overrightarrow{OA} = \begin{pmatrix} 6 \\ 3 \end{pmatrix}$

$B(0, -5) \Rightarrow \overrightarrow{OB} = \begin{pmatrix} 0 \\ -5 \end{pmatrix}$

$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$
$= \begin{pmatrix} 0 \\ -5 \end{pmatrix} - \begin{pmatrix} 6 \\ 3 \end{pmatrix}$
$= \begin{pmatrix} \mathbf{-6} \\ \mathbf{-8} \end{pmatrix}$

(b) $\left|\overrightarrow{AB}\right| = \sqrt{(-6)^2 + (-8)^2}$
$= \sqrt{100}$
$= 10$

Unit vector in the direction of $\overrightarrow{AB}$
$= \frac{1}{\left|\overrightarrow{AB}\right|}(\overrightarrow{AB})$
$= \frac{1}{10}\begin{pmatrix} -6 \\ -8 \end{pmatrix}$
$= \begin{pmatrix} \mathbf{-\frac{3}{5}} \\ \mathbf{-\frac{4}{5}} \end{pmatrix}$.

9 (a) Since the point X, Y and Z are collinear,
$\overrightarrow{XY} = k\overrightarrow{YZ}$
$2\underline{a} + 4\underline{b} = k(3\underline{a} + m\underline{b})$
$2\underline{a} + 4\underline{b} = 3k\underline{a} + km\underline{b}$

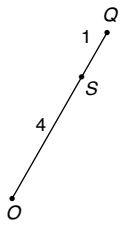
Equating the coefficients of $\underline{a}$,
$3k = 2$
$k = \frac{2}{3}$

Equating the coefficients of $\underline{b}$,
$km = 4$
$\frac{2}{3}m = 4$
$m = \mathbf{6}$

$k = \frac{2}{3}$

(b) $\overrightarrow{XY} = \frac{2}{3}\overrightarrow{YZ}$
$\left|\overrightarrow{XY}\right| = \frac{2}{3}\left|\overrightarrow{YZ}\right|$
$\frac{\left|\overrightarrow{XY}\right|}{\left|\overrightarrow{YZ}\right|} = \frac{2}{3}$
$XY : YZ = \mathbf{2 : 3}$

10 $OS = 4SQ$
$\frac{OS}{SQ} = 4$
$OS : SQ = 4 : 1$



$\therefore \overrightarrow{OS} = \frac{4}{5}\overrightarrow{OQ}$
$= \frac{4}{5}(10\underline{a} + 5\underline{b})$
$= \mathbf{8\underline{a} + 4\underline{b}}$

11 (a) $3\underline{r} - \underline{s} = 3\begin{pmatrix} 5 \\ 4 \end{pmatrix} - \begin{pmatrix} 7 \\ -3 \end{pmatrix}$
$= \begin{pmatrix} \mathbf{8} \\ \mathbf{15} \end{pmatrix}$

(b) $|3\underline{r} - \underline{s}| = \sqrt{8^2 + 15^2} = \sqrt{289}$
$= 17$

Hence, the unit vector in the direction of $3\underline{r} - \underline{s}$
$= \frac{1}{17}\begin{pmatrix} 8 \\ 15 \end{pmatrix} = \begin{pmatrix} \mathbf{\frac{8}{17}} \\ \mathbf{\frac{15}{17}} \end{pmatrix}$.

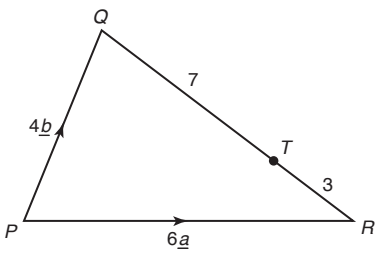
12 $(p + 4)\underline{u} = (q - 3)\underline{v}$

If $\underline{u}$ is not parallel to $\underline{v}$, then the coefficients of $\underline{u}$ and $\underline{v}$ are both zero.

$p + 4 = 0$ $\qquad$ $q - 3 = 0$

$p = \mathbf{-4}$ $\qquad$ $q = \mathbf{3}$

13



(a) $\overrightarrow{QR} = \overrightarrow{QP} + \overrightarrow{PR}$

$= \mathbf{-4\underline{b} + 6\underline{a}}$

(b) $\overrightarrow{PT} = \overrightarrow{PQ} + \overrightarrow{QT}$

$= \overrightarrow{PQ} + \frac{7}{10}\overrightarrow{QR}$

$= 4\underline{b} + \frac{7}{10}(-4\underline{b} + 6\underline{a})$

$= 4\underline{b} - \frac{14}{5}\underline{b} + \frac{21}{5}\underline{a}$

$= \mathbf{\frac{6}{5}\underline{b} + \frac{21}{5}\underline{a}}$

Paper 2

1 (a) Using the concept of position vectors:

$$\overrightarrow{AB} = \begin{pmatrix}10\\14\end{pmatrix}$$

$$\overrightarrow{OB} - \overrightarrow{OA} = \begin{pmatrix}10\\14\end{pmatrix}$$

$$\begin{pmatrix}4\\6\end{pmatrix} - \overrightarrow{OA} = \begin{pmatrix}10\\14\end{pmatrix}$$

$$\overrightarrow{OA} = \begin{pmatrix}4\\6\end{pmatrix} - \begin{pmatrix}10\\14\end{pmatrix}$$

$$\overrightarrow{OA} = \begin{pmatrix}-6\\-8\end{pmatrix}$$

$\therefore A = \mathbf{(-6, -8)}$

(b) $\left|\overrightarrow{OA}\right| = \sqrt{(-6)^2 + (-8)^2} = \sqrt{100} = 10$

Unit vector in the direction of $\overrightarrow{OA}$

$$= \frac{1}{\left|\overrightarrow{OA}\right|}(\overrightarrow{OA})$$

$$= \frac{1}{10}\begin{pmatrix}-6\\-8\end{pmatrix}$$

$$= \begin{pmatrix}-\mathbf{\frac{3}{5}}\\-\mathbf{\frac{4}{5}}\end{pmatrix}$$

(c) Since $\overrightarrow{CD}$ is parallel to $\overrightarrow{AB}$, $\overrightarrow{CD} = k\overrightarrow{AB}$ (k is a constant).

$$\begin{pmatrix}m\\7\end{pmatrix} = k\begin{pmatrix}10\\14\end{pmatrix}$$

$$\begin{pmatrix}m\\7\end{pmatrix} = \begin{pmatrix}10k\\14k\end{pmatrix}$$

Equating the x-components:

$m = 10k \ldots$ (1)

Equating the y-components:

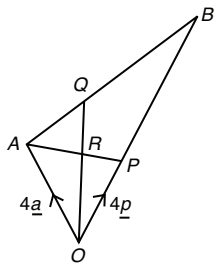
$7 = 14k$

$k = \frac{1}{2}$

From (1), when $k = \frac{1}{2}$,

$m = 10\left(\frac{1}{2}\right) = \mathbf{5}$

2



(a) (i) $\overrightarrow{AP} = \overrightarrow{AO} + \overrightarrow{OP}$

$\overrightarrow{AP} = \mathbf{-4\underline{a} + 4\underline{p}}$

(ii) $\overrightarrow{OQ} = \overrightarrow{OA} + \overrightarrow{AQ}$

$\overrightarrow{OQ} = \overrightarrow{OA} + \frac{1}{4}\overrightarrow{AB}$

Given $\overrightarrow{AQ} = \frac{1}{4}\overrightarrow{AB}$

$\overrightarrow{OQ} = \overrightarrow{OA} + \frac{1}{4}(\overrightarrow{AO} + \overrightarrow{OB})$

$\overrightarrow{OQ} = \overrightarrow{OA} + \frac{1}{4}(\overrightarrow{AO} + 3\overrightarrow{OP})$

Given that $\overrightarrow{OP} = \frac{1}{3}\overrightarrow{OB}$, $\therefore \overrightarrow{OB} = 3\overrightarrow{OP}$.

$\overrightarrow{OQ} = 4\underline{a} + \frac{1}{4}[-4\underline{a} + 3(4\underline{p})]$

$\overrightarrow{OQ} = 4\underline{a} - \underline{a} + 3\underline{p}$

$\overrightarrow{OQ} = \mathbf{3\underline{a} + 3\underline{p}}$

(b) (i) $\overrightarrow{AR} = h\overrightarrow{AP} = h(-4\underline{a} + 4\underline{p})$

$\overrightarrow{AR} = \mathbf{-4h\underline{a} + 4h\underline{p}}$

(ii) $\overrightarrow{RQ} = k\overrightarrow{OQ} = k(3\underline{a} + 3\underline{p})$

$\overrightarrow{RQ} = \mathbf{3k\underline{a} + 3k\underline{p}}$

(c) $\overrightarrow{AQ} = \overrightarrow{AR} + \overrightarrow{RQ}$

$-\underline{a} + 3\underline{p} = -4h\underline{a} + 4h\underline{p} + 3k\underline{a} + 3k\underline{p}$

$-\underline{a} + 3\underline{p} = (-4h + 3k)\underline{a} + (4h + 3k)\underline{p}$

From (a)(ii), $\overrightarrow{AQ} = -\underline{a} + 3\underline{p}$

Equating the coefficients of $\underline{a}$ and $\underline{b}$ respectively, we have:

$$\begin{array}{rl} -4h + 3k = -1 & \ldots (1) \\ (+)\quad 4h + 3k = 3 & \ldots (2) \\ \hline 6k = 2 & \\ k = \mathbf{\frac{1}{3}} & \end{array}$$

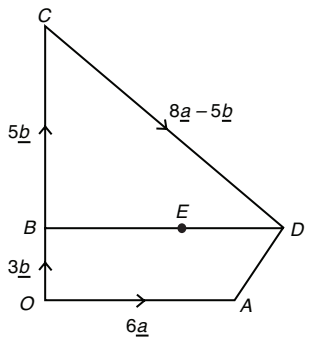
From (1): $-4h + 3\left(\frac{1}{3}\right) = -1$

$$-4h = -2$$

$$h = \mathbf{\frac{1}{2}}$$

3 (a)

Given that $OB = \frac{3}{8}OC$, thus $OC = \frac{8}{3}OB$.
Therefore, $\overrightarrow{OC} = \frac{8}{3}\overrightarrow{OB} = \frac{8}{3}(3\underline{b}) = 8\underline{b}$.
This gives $\overrightarrow{BC} = 8\underline{b} - 3\underline{b} = 5\underline{b}$.



(i) $\overrightarrow{AC} = \overrightarrow{AO} + \overrightarrow{OC}$
$= \mathbf{-6\underline{a} + 8\underline{b}}$

(ii) $\overrightarrow{BD} = \overrightarrow{BC} + \overrightarrow{CD}$
$= 5\underline{b} + (8\underline{a} - 5\underline{b})$
$= \mathbf{8\underline{a}}$

(b) If the points A, E and C are collinear, then

$$\overrightarrow{AC} = k\overrightarrow{AE}$$

$$-6\underline{a} + 8\underline{b} = k(\overrightarrow{AO} + \overrightarrow{OB} + \overrightarrow{BE})$$

$$-6\underline{a} + 8\underline{b} = k(-6\underline{a} + 3\underline{b} + h\overrightarrow{BD})$$

$$-6\underline{a} + 8\underline{b} = k[-6\underline{a} + 3\underline{b} + h(8\underline{a})]$$

$$-6\underline{a} + 8\underline{b} = -6k\underline{a} + 3k\underline{b} + 8hk\underline{a}$$

$$-6\underline{a} + 8\underline{b} = (-6k + 8hk)\underline{a} + 3k\underline{b}$$

Equating the coefficients of $\underline{b}$:

$$3k = 8$$

$$k = \frac{8}{3}$$

Equating the coefficients of $\underline{a}$:

$$-6 = -6k + 8hk$$

$$-6 = -6\left(\frac{8}{3}\right) + 8h\left(\frac{8}{3}\right)$$

$$-18 = -48 + 64h$$

$$30 = 64h$$

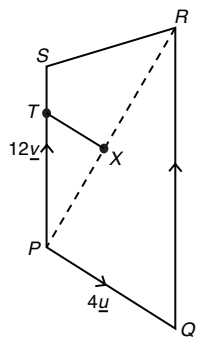
$$h = \mathbf{\frac{15}{32}}$$

(c) $\left|\overrightarrow{OC}\right| = |8\underline{b}| = 8|\underline{b}| = 8(2) = 16$

$\left|\overrightarrow{OA}\right| = |6\underline{a}| = 6|\underline{a}| = 6(2) = 12$

$\left|\overrightarrow{AC}\right| = \sqrt{\left|\overrightarrow{OC}\right|^2 + \left|\overrightarrow{OA}\right|^2}$
$= \sqrt{16^2 + 12^2}$
$= \mathbf{20}$

4



(a) $\overrightarrow{PR} = \overrightarrow{PQ} + \overrightarrow{QR}$
$= 4\underline{u} + \frac{3}{2}\overrightarrow{PS}$
$= 4\underline{u} + \frac{3}{2}(12\underline{v})$
$= \mathbf{4\underline{u} + 18\underline{v}}$

Given that $\overrightarrow{PS} = \frac{2}{3}\overrightarrow{QR}$, thus $\overrightarrow{QR} = \frac{3}{2}\overrightarrow{PS}$.

(b) (i) $\overrightarrow{TX} = m\overrightarrow{PQ}$
$= m(4\underline{u})$
$= \mathbf{4m\underline{u}}$

(ii) P, X and R are collinear.

$$\overrightarrow{PX} = k\overrightarrow{PR}$$

$$\overrightarrow{PT} + \overrightarrow{TX} = k(4\underline{u} + 18\underline{v})$$

$$\frac{3}{4}\overrightarrow{PS} + 4m\underline{u} = 4k\underline{u} + 18k\underline{v}$$

Given From (b) (i)

$$\frac{3}{4}(12\underline{v}) + 4m\underline{u} = 4k\underline{u} + 18k\underline{v}$$

$$9\underline{v} + 4m\underline{u} = 4k\underline{u} + 18k\underline{v}$$

Equating the coefficients of $\underline{v}$,

$9 = 18k$

$k = \frac{1}{2}$

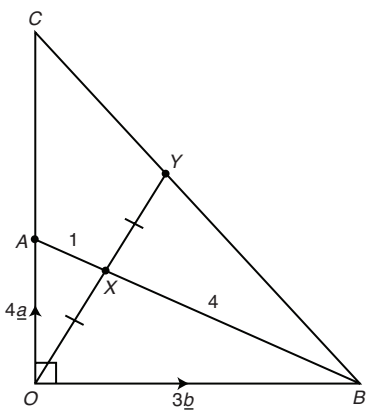
Equating the coefficients of $\underline{u}$,

$4m = 4k$

$4m = 4\left(\frac{1}{2}\right)$

$m = \mathbf{\frac{1}{2}}$

5



(a) (i) $\overrightarrow{OX} = \overrightarrow{OA} + \overrightarrow{AX}$

$= 4\underline{a} + \frac{1}{5}\overrightarrow{AB}$

$= 4\underline{a} + \frac{1}{5}(\overrightarrow{AO} + \overrightarrow{OB})$

$= 4\underline{a} + \frac{1}{5}(-4\underline{a} + 3\underline{b})$

$= 4\underline{a} - \frac{4}{5}\underline{a} + \frac{3}{5}\underline{b}$

$= \mathbf{\frac{16}{5}\underline{a} + \frac{3}{5}\underline{b}}$

(ii) $\overrightarrow{BY} = \overrightarrow{BO} + \overrightarrow{OY}$

$= -3\underline{b} + 2\overrightarrow{OX}$

$= -3\underline{b} + 2\left(\frac{16}{5}\underline{a} + \frac{3}{5}\underline{b}\right)$

$= -3\underline{b} + \frac{32}{5}\underline{a} + \frac{6}{5}\underline{b}$

$= \mathbf{\frac{32}{5}\underline{a} - \frac{9}{5}\underline{b}}$

(b) $\overrightarrow{BC} = k\overrightarrow{BY}$

$\overrightarrow{BO} + \overrightarrow{OC} = k\overrightarrow{BY}$

$\overrightarrow{BO} + h\overrightarrow{OA} = k\overrightarrow{BY}$

$-3\underline{b} + h(4\underline{a}) = k\left(\frac{32}{5}\underline{a} - \frac{9}{5}\underline{b}\right)$

$-3\underline{b} + 4h\underline{a} = \frac{32}{5}k\underline{a} - \frac{9}{5}k\underline{b}$

Comparing the coefficients of $\underline{b}$,

$-\frac{9}{5}k = -3$

$k = 3 \times \frac{5}{9}$

$k = \mathbf{\frac{5}{3}}$

Comparing the coefficients of $\underline{a}$,

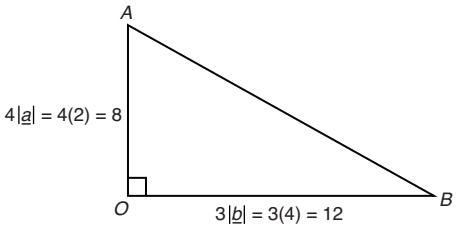
$4h = \frac{32}{5}k$

$4h = \frac{32}{5} \times \frac{5}{3}$

$h = \frac{32}{5} \times \frac{5}{3} \times \frac{1}{4}$

$h = \mathbf{\frac{8}{3}}$

(c)



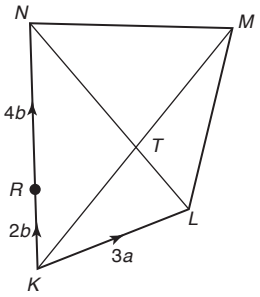
Using the Pythagoras' theorem,

$\left|\overrightarrow{AB}\right| = \sqrt{8^2 + 12^2}$

$= \sqrt{208}$

$=$ **14.42 units**

6



(a) (i) Given $\overrightarrow{KR} = \frac{1}{3}\overrightarrow{KN}$, then

$$\overrightarrow{KN} = 3\overrightarrow{KR}$$
$$\overrightarrow{KN} = 3(2\underline{b})$$
$$\overrightarrow{KN} = 6\underline{b}$$

$$\overrightarrow{NL} = \overrightarrow{NK} + \overrightarrow{KL}$$
$$= \mathbf{-6\underline{b} + 3\underline{a}}$$

(ii) $\overrightarrow{KT} = \overrightarrow{KL} + \overrightarrow{LT}$

$$= \overrightarrow{KL} + \frac{1}{3}\overrightarrow{LN}$$
$$= 3\underline{a} + \frac{1}{3}(6\underline{b} - 3\underline{a})$$
$$= 3\underline{a} + 2\underline{b} - \underline{a}$$
$$= \mathbf{2\underline{a} + 2\underline{b}}$$

(b) (i) $\overrightarrow{NM} = \overrightarrow{NK} + \overrightarrow{KM}$

$$= \overrightarrow{NK} + \frac{1}{q}\overrightarrow{KT}$$
$$= -6\underline{b} + \frac{1}{q}(2\underline{a} + 2\underline{b})$$
$$= -6\underline{b} + \frac{2}{q}\underline{a} + \frac{2}{q}\underline{b}$$
$$= \mathbf{\left(\frac{2}{q} - 6\right)\underline{b} + \frac{2}{q}\underline{a}}$$

(ii) But it is given that $\overrightarrow{NM} = 3p\underline{a} - 2\underline{b}$.

By comparison,

$$\frac{2}{q} - 6 = -2$$
$$\frac{2}{q} = 4$$
$$q = \mathbf{\frac{1}{2}}$$

and

$$3p = \frac{2}{q}$$
$$p = \frac{2}{3q}$$
$$p = \frac{2}{3\left(\frac{1}{2}\right)}$$
$$p = \mathbf{\frac{4}{3}}$$